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Theoretical Modeling of Pseudo Hydrostatic Force in Solid-Liquid Pipe Flow with Two Layers

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ABSTRACT

In the moving layer of particles with variable concentration, the shear estimation is not directly predictable and there is no existing clear mathematical or empirical formula to achieve this objective. This paper presents a developed approach to estimate the shear forces in a flow having suspended and moving layers of solid particles in liquid flow. The two-layer approach was taken whereby the flow consisting of one upper suspended layer of particles in the liquid, and the bottom layer was the moving bed of particles. In the present work, the method of finding the force acting on the pipe wall by the particles in the layer, termed as the 'dry force', was presented using a "pseudo hydrostatic pressure" method. To attain the equation for the dry force, a mathematical approach is taken with the assumptions that the flow is horizontal, two-phase pipe flow (solid in Newtonian liquid), incompressible and it is at steady-state. The analysis was conducted considering various particles densities, various concentrations in the suspended layer and different thicknesses of the moving bed. Changing the concentration in the suspended layer from 0.00001 up to 0.001 didn't showed significant changes in the dry force evaluation. The dry friction force is increasing with increasing moving bed thickness. The developed mathematical model can be applicable in solving for the shear force in horizontal solid liquid two-phase flows.

Keywords: Pseudo Hydrostatic, two phase flow, transport phenomena,

INTRODUCTION

In the study of solid-in-liquid two-phase flows, shear stress is an important parameter in determining the frictional forces that are acting on the pipe wall. In case of homogenous solid-in-liquid suspension flow, the properties can be treated as mixture properties with constant concentration profile across the flow area, which is not possible in the case of variable concentration profile, where two types of two-phase flow layers appear in the flow.

The solid-in-liquid flows are complex to be modeled, and due to this, the suspended layer is usually treated as a single-phase fluid

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with modified properties which depends on the solids concentration (Crowe *et al.,* 1998). When the concentration is significantly differ, the two-layer approach was taken whereby the flow consists of one upper suspended layer of particles in the fluid, and the bottom layer was the moving bed of particles, as in fig.1.

The flow of solid–liquid mixtures in conduits is encountered in several situations of industrial significance like ore transportation with long pipelines, oil well and geothermal drilling, mineral and waste water processing. The flow geometry may be pipe or annulus in vertical, inclined or horizontal orientation. While the issues dealing with vertical configurations have been solved after many years of research, there are several problems and questions to be answered for the flow of two phase solid–liquid mixtures in horizontal and inclined conduits (Kelessidis & Bandelis, 2005).

The concept of dispersive layer has been employed by Ramadan *et al.* (2005) to extend the two-layer modeling to a three-layer scheme. Their model considered the existence of a dispersive layer, which is sandwiched between the suspended layer and a dead bed layer. The dispersed layer was considered to have a higher concentration gradient compared to the suspended layer (Fig.2).

To solve for the shear force, Ramadan *et al.* (2005) has adopted the pseudo hydrostatic approach. Thy proposed the following equation to estimate the dry force applied by cutting particles on the pipe wall during the transportation of the drilling cuttings.

$$F_d = g\mu_d\left(\rho_p - \rho_f\right)c_d S_d t_d \sin\beta \cos\left(\frac{\left(\theta_b + \theta_d\right)}{2}\right) \qquad [1]$$

The objective of the present work is to apply the pseudo hydrostatic pressure approach to estimate the shearing force between the conduit wall and the solid in liquid flow. Material balance equations of the two phases and momentum equations of the two layers are combined to develop the model. Additional equations are introduced to estimate the average concentration of the suspended layer, and thickness and velocity of the dispersed layer. The thickness of the dispersed layer is modeled using the pseudo hydrostatic pressure gradient concept and assuming linearly varying particle concentration in the dispersed layer.

In the analysis, water as the liquid phase and two different density values of solid particles were considered. Various concentrations in the suspended layer were assumed, and the dry force results were evaluated at different thicknesses of the moving layer.

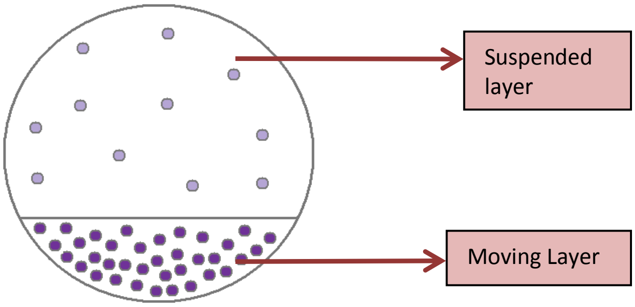

Fig.1: Solid in liquid flow with suspended layer on top of moving layer.

PHYSICAL DESCRIPTION OF TWO LAYER SOLID-LIQUID FLOW

In the present work, the flow of solid-in-liquid in pipes was divided into two layers which are:

i. The upper layer: Homogeneous Suspended Layer.

ii. The lower layer: Moving Bed Layer.

In the top layer or the suspended layer, the concentration profile is considered as homogeneous, having a constant concentration profile. This is because; there is only a small variation in its concentration, (Fig.3b), which could be neglected and the profile of the suspended layer concentration, is constant

$$\frac{dC_s}{dy} = 0 \tag{2}$$

while the moving bed has a linearly increasing concentration profile.

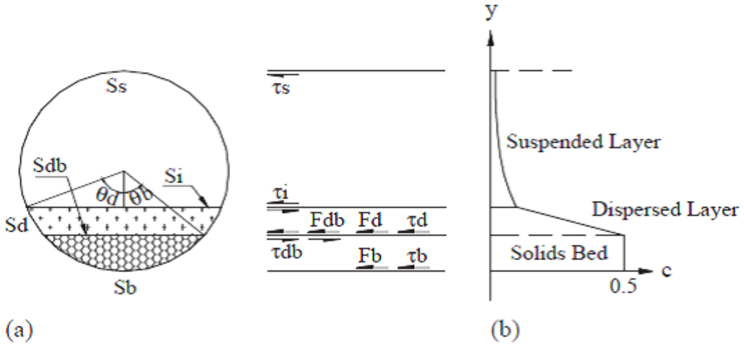

Fig.2: (a) Schematic representation of shear stresses acting in the three-layer mechanics model; and (b) assumed concentration profiles in three-layer modeling scheme (Ramadan *et al., 2005*)

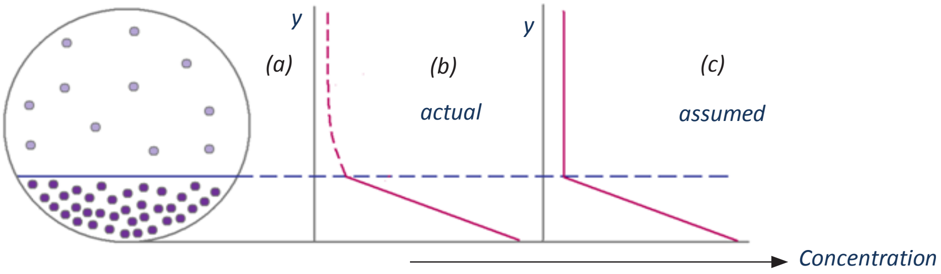

Fig.3: (a) The two-layer approach with the suspended region and the moving bed, (b) the concentration profile for suspended layer shown in dashed line and (c) concentration profile of suspended layer assumed to be homogeneous while concentration profile of the moving bed is linear

In a three-layer model, there is an additional layer at the bottom of the flow. This layer which is called dead bed or stationary bed has a maximum concentration. By both experimental and statistical methods, the bed concentration is found to have the range value (0.4805-0.52) (Cho, 2001). Therefore in this two-layer model, the maximum concentration of the moving bed is taken as 0.5, which is at the bottom of the pipe. The following have been assumed

- The flow is a two-phase pipe flow (solid-liquid)
- The flow is in horizontal pipe
- The fluid is taken as Newtonian fluid
- Two-layer approach is applied
    - Upper layer is the homogeneous suspended layer
    - Lower layer is the moving bed layer with linear concentration profile
- No-slip condition between the two layers which neglects the interstitial shear force between the two layers
- The flow is incompressible and at steady state
- Analysis is made per unit length basis (flow properties is constant in the horizontal direction)

DERIVATION OF THEORETICAL MODEL

The prediction of forces in two phase flow with multi layers requires prediction of the flow areas, the densities of the different layers, the concentration profiles in each layer, and the structure of forces created by each phase and how it applies on the conduits boundaries.

A. Forces

The total force, F_w acting on the pipe wall boundaries is the summation of the forces acting on the wall in contact with the upper suspended layer, F_{sw} and the wall of the lower moving bed, F_{mw}. It can be given by:

$$F_w = F_{sw} + F_{mw} \tag{3}$$

The average particle concentration in the suspended layer, c_s is very small compared to the average concentration of the particles in the moving bed layer $C_s << C_m$ (Newitt *et al*., 1955). Therefore, the force acting on the upper wall only comes from the shear between the homogeneous solid-liquid flow (of mixed density) and the pipe wall:

$$F_{sw} = \tau_{sw} A_s \tag{4}$$

The moving bed layer has a higher concentration of particles which will exert additional force. This force is the dry friction force, F_d that is acted by the particles in the moving bed layer upon the bottom wall boundaries, S_{mw}. The force between the moving bed and wall, F_{mw} becomes:

$$F_{mw} = (\tau_{mw} \ A_m + F_d) \qquad [5]$$

This frictional force between the moving bed layer and the wall will be determined using the pseudo hydrostatic pressure distribution on the wall and will be analysed per unit length basis.

B. Flow Area

By simplification of considering unit length basis, the area between each layer and the contact wall becomes the wetted perimeter between them:

$$A_s = S_{sw} \ x \ 1 \text{ unit length} \qquad [6]$$

$$A_m = S_{sw} \ x \ 1 \text{ unit length} \qquad [7]$$

C. Density

According to the two phase flow assumption, the density of each of the two layers will be the mixed densities between the fluid and solids phases according to the solid concentrations in each layer. The density of the fluid phase ρ_f depends solely on the properties of fluid used. Meanwhile, the density of particles depends on both particle properties ρ_s and particles volumetric concentration c_i in the layer. This can be expressed by the following relation:

$$\rho_s = c_s \ \rho_p + (1 - c_s) \rho_f \qquad [8]$$

$$\rho_m = c_m \ \rho_p + (1 - c_m) \rho_f \qquad [9]$$

where, ρ_s and ρ_m are the densities of the mixture in the suspended layer and the mean density of the moving layer, respectively.

D. Dry Friction Force

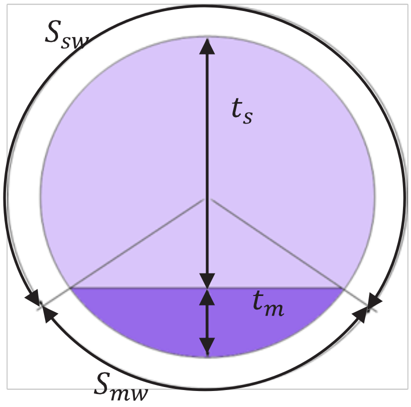

Fig.4: Thickness and Perimeter of each layer in determining the pseudo hydrostatic pressure

To get the dry friction force F_d on moving bed wall, the pseudo hydrostatic pressure approach shall be used. Following the simple definition of the pseudo hydrostatic pressure distribution on the moving bed boundary, the pressure can be estimated as total force acting on that boundary per the area of wall in contact with the moving bed region for one unit length:

$$p_{Pseudo} = F / A_w \tag{10}$$

The dynamic friction coefficient between particles and channel wall is μ_d. Then the dry friction force will be written as:

$$F_d = \mu_d P_{Pseudo} A_m \tag{11}$$

$$p_{Pseudo} = p_m = \rho_m \cdot g \cdot \tag{12}$$

E. The Pseudo Hydrostatic Pressure

Based on the pseudo hydrostatic pressure concept, the hydrostatic pressure distribution along the moving bed wall can be defined as:

$$p_{Pseudo} = \int_0^{t_m} \left[\rho_p c_m + \rho_f (1 - c_m)\right] g\ t$$

$$p_{Pseudo} = \int_0^{t_m} \rho_P c_m g\ dt + \int_0^{t_m} \rho_f \left(1 - c_m\right) g\ dt \tag{13}$$

F. Concentration

The average particles volumetric concentration in the suspended layer is very small compared with the moving-bed layer. Thus we assume that the concentration profile is constant. Fredsoe and Deigaard (1992) suggested the assumption of linear variation for the dispersed layer. By adopting the pseudo hydrostatic gradient, the average concentration of the moving-bed layer can be approximated as follows:

$$c = c_o - \frac{z}{\delta_s}\left(c_o - c_\delta\right) \tag{14}$$

In the equation, c_δ is concentration at the top of the sheet layer and c_o is the maximum concentration. In our case, $c_\delta = c_s$ where at the interface of the suspended and moving layers, the concentration is equal. The maximum concentration is taken as the concentration at the bottom of the moving bed layer, therefore $c_o = c_{m,\max}$. Hence, using the notations of fig.4, the following relation is obtained

$$c = c_{m,\max} - \frac{t}{t_m}\left(c_{m,\max} - c_s\right) \tag{15}$$

where, t is a height in the moving bed, and t_m is the maximum height of the moving bed.

Substituting Equation [15] into Equation [13], and integrating to find P_m:

$$p_m = \int_0^{t_m} \rho_p g \left[c_{m,\max} - \frac{t}{t_m}\left(c_{m,\max} - c_s\right) \right] dt + \int_0^{t_m} \rho_f g \left(1 - \left[c_{m,\max} - \frac{t}{t_m}\left(c_{m,\max} - c_s\right) \right] \right) g\ dt$$

$$p_m = \rho_p g \left[t c_{m,\max} - \frac{t^2}{2t_m}\left(c_{m,\max} - c_s\right) \right]_0^{t_m} + \rho_f g \left[t - t c_{m,\max} + \frac{t^2}{2t_m}\left(c_{m,\max} - c_s\right) \right]_0^{t_m}$$

Leading to:

$$p_m = \rho_p g t_m \left(\frac{\left(c_{m,max} + c_s\right)}{2} \right) + \rho_f g t_m \left(1 - \frac{\left(c_{m,\max} + c_s\right)}{2} \right) \qquad [16]$$

Combining Equation [16] with equation [11], the moving layer dry friction force per unit length becomes:

$$F_d = \mu_d \left[\rho_p \left(\frac{\left(c_{m,\max} + c_s\right)}{2} \right) + \rho_f \left(1 - \frac{\left(c_{m,\max} + c_s\right)}{2} \right) \right] g t_m S_{mw} \qquad [17]$$

COMPUTATION PROCEDURE AND RESULTS

To test the validity of the developed model, a calculation program is created using Microsoft Excel, including all inputs and desired outputs to be calculated. The enveloped equation in this work, equation 14, to estimate the shear drag force is programmed. Also, equation 1 suggested by Ramadan *et al.* (2005) is programmed so that they are analysing input data simultaneously.

The selected materials for the present analysis are water and cutting particles from oil well drilling site. Same parameters used by Ramadan *et al.* (2005) were adopted here. The properties of the liquid phase are shown in table 1.

TABLE 1

Constant computational data of the liquid phase

Density of water	1000 kg/m^3
Viscosity of water	0.001 Pa.s
Channel diameter	70 mm
Dynamic friction factor	0.25

For the solid phase, the particles are selected with mean diameter of 3.8x10^4 m. To standardize the calculations, initial concentration for suspended layer, c_s is assumed to be relatively small, = 0.00001. Two values of particle density are used in the iteration, 1922 kg/m^3 and 2600 kg/m$^{3.}$ The calculations were made at different thicknesses of the moving bed layer which is considered as the variable pre-set parameter. The values were varied from 0.005 m to 0.020 m in steps of 0.0025 m, to solve for the dry force, F_d. For the entire analysis, the total concentration of the solid in liquid flow is fixed at 0.08 m^3/m^3.

As studied by Nguyen (1999) the dynamic friction coefficient was approximated to be half of the static with around 0.2 values. This value of the friction coefficient seems high, but should bear in mind that this is for solid-in-liquid two-phase flow. The dynamic friction coefficient was selected here as 0.25.

DISCUSSION OF RESULTS

The predicted dry forces at very low concentration in the suspended layer (C_s=0.000001), with different densities, are shown in figures 11a and 11b. For tested particles densities of 2600 kg/m^3, as in fig.5, and 1922 kg/m^3, as in fig.6, the results of the developed model shows higher values compared to the dry forces predicted by using Ramadan *et al.* (2005) model. No significant changes could be noticed between the two density cases in the dry force values from the present model. In contrast, the prediction of the dry forces based on Ramadan *et al.* (2005) model shows reduction in the values of the dry forces as the density reduced. This reduction becomes more significant when the moving bed layer is increased. At moving bed height of 0.02 m, the reduction is about 12% while for Ramadan *et al.* (2005) model, the reduction is 42%. This indicates that Ramadan *et al.* (2005) model is more sensitive to the changes in densities of the particles.

Prediction of the dry forces at high suspended layer concentration for particle density 2600 kg/m^3 and 1922 kg/m^3 are shown in fig.7 and fig.8, respectively. Similar to the case of low concentration, the reduction in the dry friction value is 12.1%, as prediction results from the developed model. Comparing this with ref. Ramadan *et al.* (2005) model, the reduction is 42.4%. This demonstrates that as the particle density reduced, the dry friction forces created by the solids on the contact walls of the pipe are reduced, correspondingly.

To examine the effect of the suspended layer concentration, predicted dry forces values are shown in table 2 as predicted by the present mathematical model and ref. Ramadan *et al.*

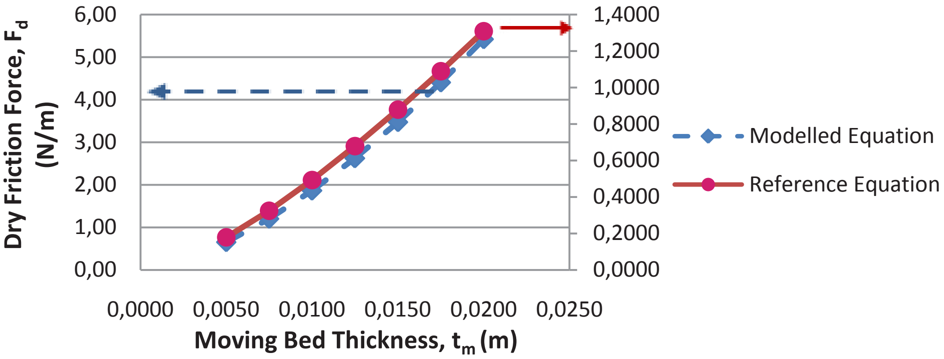

Fig.5: Dry Friction Force vs. Moving Bed Thickness, C_s=0.000001, and ρ_p=2600 kg/m^3

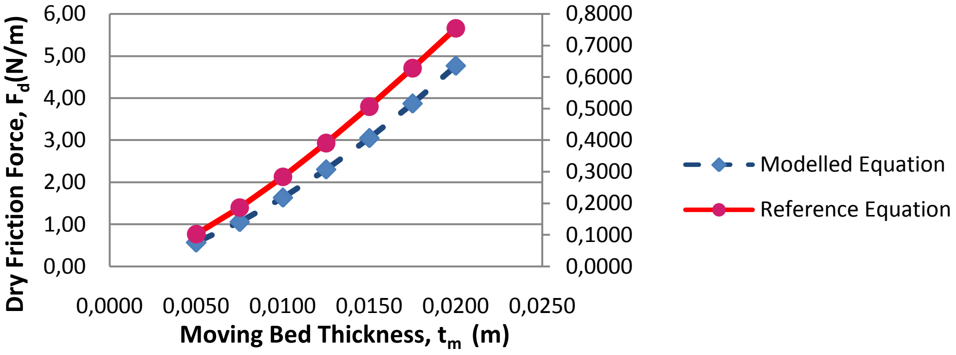

Fig.6: Dry Friction Force vs. Moving Bed Thickness, C_s=0.000001, and ρ_p=1922 kg/m^3

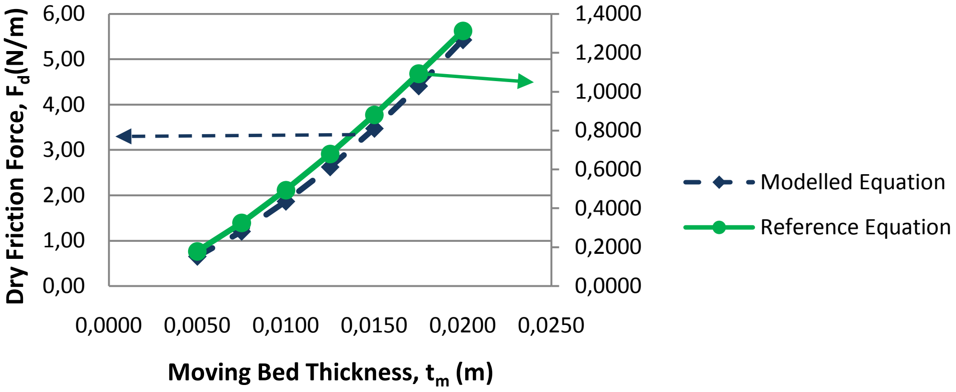

Fig.7: Dry Friction Factor vs. Moving Bed Thickness C_s=0.001, and ρ_p=2600 kg/m^3

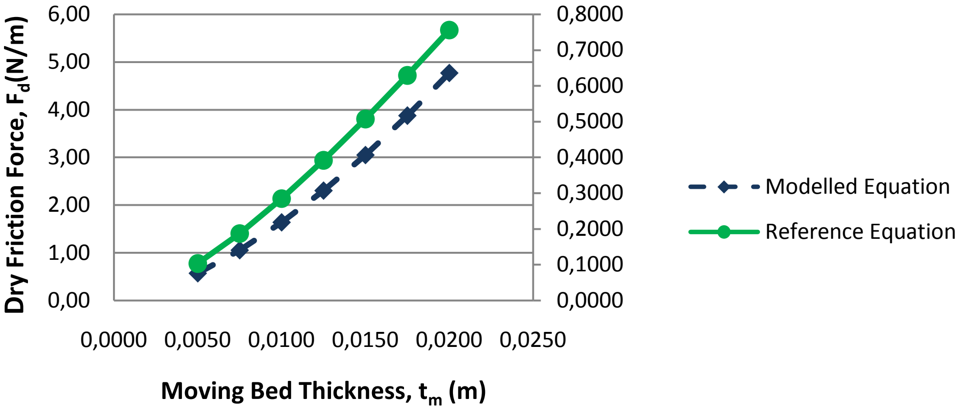

Fig.8: Dry Friction Factor vs. Moving Bed Thickness, C_s=0.001, and ρ_p=1922 kg/m^3

(2005) model. The results are predicted at two different concentrations, low concentration of 0.00001 m^3/m^3, and high concentration of 0.001 m^3/m^3. It can be noticed that the change of the suspended layer concentration does not affect the dry frictional forces considerably. This is noted in both, the recent model, and Ramadan *et al.* (2005) model. At moving bed thickness of 0.02 m, the increase of concentration from low to high mentioned values cause the dry friction force to increased by 0.06% based on the present model, while it increased by 0.2% based on Ramadan *et al.* (2005) model. This is because the concentration of the suspended layer is always much smaller than that of the moving bed. Therefore, any change in its value, provided still agreeing with the assumption of ($C_s << C_m$), does not contribute to a high increment in the dry friction force.

It can be seen that for both modelled equation and reference equation, the dry friction force is increasing with increasing moving bed thickness. However, the reference equation gives a much smaller value of predicted dry force compared with results of the developed model in this paper. The difference is highly significant. However, it can be justified by the following explanations:

TABLE 2
Predicted dry friction force values at different concentrations (per unit lenght of the pipe)

	Modelled Equation		**Reference Equation**	
c_s m^3/m^3	0.00001	0.001	0.00001	0.001
t_m (m)	F_d (N/m)			
0.0050	0.6503	0.6505	0.1790	0.1794
0.0075	1.2022	1.2029	0.3246	0.3252
0.0100	1.8631	1.8642	0.4928	0.4938
0.0125	2.6215	2.623	0.6788	0.6802
0.0150	3.4702	3.4721	0.8789	0.8806
0.0175	4.4045	4.4071	1.0899	1.092
0.0200	5.4217	5.4247	1.3092	1.3118

i. Ramadan *et al.* (2005) equation is built for a three-layer application. The assumptions made in developing the equation may only be suited to three-layer flows.

ii. In their equation, the authors Ramadan *et al.* (2005) have considered the average angular distance between the dispersed layer and the solids bed layer (Fig.2). This angular average might be insignificant when the equation is applied to a two-layer model, where the value of $\boldsymbol{\theta_b}$ will be zero.

iii. The modelled equation finds the dry friction force acting on the pipe wall by the layer of particles. In the actual case, only particles in contact with the wall would exert dry friction force. This could mean that only a percentage of the pseudo hydrostatic force contributes to the dry friction force on the pipe wall in contact with the moving bed. For this, we can assume that if the contact between particles and lower layer pipe wall is 25% of total contact area between moving bed (fluid and particles) and pipe wall, the dry friction force could also be reduced to 25%, which could give an excellent agreement with the reference equation.

iv. The dry friction coefficient is selected as 0.25, while Ramadan *et al.* (2005) used value of 0.2.

CONCLUSION

A mathematical formula has been developed to estimate the dry friction force of a horizontal pipe solid-liquid flow using the two-layer approach. The model can be modified to match solid-liquid flow application to serve in solving the complexity of calculating the boundary-moving bed force in different types of two phase flow with multi layers of concentration. The model is useful in modelling and analysis of cutting particle transportation and sand-water sedimentation.

The basics of the calculation program have been made in Microsoft Excel. The developed mathematical model is tested against one available model that also applies the pseudo hydrostatic pressure method, using similar data (Ramadan *et al.*, 2005). Based on the calculated results, there is lack of agreement between the modelled equation and the reference equation. This difference is justified by several factors which include assumptions made for mathematical modelling and dissimilar application for different flow models (two-phase or three-phase).

There are many improvements that can be made in order to achieve more reliable results wider applications of the approach:

i. The effect of particle size and channel diameter can be included in future investigations.

ii. An experiment could be conducted to compare the mathematical models with experimental results.

iii. The search can be extended to Non-Newtonian fluids.

iv. The application of the pseudo hydrostatic pressure can be considered in inclined channels.

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